

## Mid-term exam, 7 March 2018

All aids are allowed, except a computer-like device (including tablets and smartphones) and personal assistance. The exam consists of one question with five subquestions (labeled by letters **a)-e)**, with weights as indicated) that should all be answered to achieve the maximal 15 points. Question **e)** is subdivided into three parts *i)-iii)*, with further instructions below. The subquestions and parts can be answered essentially independently of each other. The mid-term exam accounts for 15% of the course mark; however, every student may choose to waive the result of the mid-term exam. The duration of the mid-term exam is 1 hour.

Generally, **statistical models and methods should be specified**, and data should be described graphically whenever that would be useful/relevant. If you realize that you need more information than is provided to carry out the analysis, specify what information you need, how you would obtain it using Minitab/Stata, and how you would use it.

### Question 1 (15 points)

An early study examined the digestibility ( $D$ ) of raw and roasted peanuts in rats, measured as the amount (in %) of nitrogen retained in the body. The study involved 20 rats which were grouped in 10 sets so that the two rats in a set were from the same litter and of the same sex, and had approximately the same weight. Within each set, one of the rats was fed raw peanuts and the other rat was fed roasted peanuts. The observations are given below, with means and standard deviations (sd).

Set	Sex	Digestibility (in %)		
		Raw	Roasted	Raw-Roasted
1	f	97.7	95.8	1.9
2	f	99.2	98.0	1.2
3	f	96.1	94.4	1.7
4	f	95.9	96.8	-0.9
5	m	97.2	97.1	0.1
6	m	99.5	96.2	3.3
7	m	98.6	96.5	2.1
8	m	97.2	95.0	2.2
9	m	96.6	95.0	1.6
10	m	95.9	96.7	-0.8
total female	f	mean = 97.23 sd = 1.544	mean = 96.25 sd = 1.526	mean = 0.975 sd = 1.284
total male	m	mean = 97.50 sd = 1.324	mean = 96.08 sd = 0.889	mean = 1.417 sd = 1.504
total overall	f+m	mean = 97.39 sd = 1.337	mean = 96.15 sd = 1.106	mean = 1.240 sd = 1.363

For example, in Set no. 3 both rats were females, and the rat that was fed raw peanuts had a digestibility of  $D = 96.1$  whereas the rat that was fed roasted peanuts had  $D = 94.4$ . Also, the mean and standard deviation for the female rats that received raw peanuts were 97.23 and 1.544, respectively, and the overall mean and standard deviation for the rats that were fed raw peanuts were 97.39 and 1.337. All values given are in percent (%).

Use the data and the descriptive statistics provided, as well as the attached Minitab (Stata) listings, to answer the questions below.

- a) (2 points) Characterize the study type (e.g., experimental or another type) and design (e.g., one-sample or another design). Discuss briefly, e.g. by using specific examples, which observations may be considered independent of each other and which observations (if any) should not be considered independent of each other.
- b) (4 points) The primary purpose of the study was to compare the digestibility of the two types of peanuts. With this purpose in mind, formulate an appropriate statistical model and analyse the data to assess whether such a difference seems to exist, and quantify its magnitude by suitable 90% and 95% intervals. Formulate your conclusion both statistically and in a less technical version that could be used as a valid summary for a non-statistical readers audience.
- c) (3 points) After having carried out the analysis of **b)**, the researchers worried that a (potential) difference in digestibility might not be the same in female and male rats. (It would also be possible to consider this question prior to the analysis in **b)**). Formulate an appropriate statistical model for this purpose, carry out a statistical test to investigate the issue, and draw conclusions.
- d) (2 points) The researchers also wanted to compare the digestibility of raw peanuts in female and male rats. Irrespective of your results in previous questions, formulate an appropriate statistical model for this purpose, carry out a statistical test to perform the comparison, and draw conclusions (possibly incorporating your findings from **b)** and **c)** as well).
- e) (4 points) For this question, you should answer **two of the three parts i)-iii)** below, each with the same score. It is allowed (but not recommended) to answer all parts, in which case your score for **e)** will be for the best two parts.
  - i) (2 points) Due to the small data set and the difficulty of assessing normality assumptions, the researchers wanted to confirm their findings for the main question of the study (as described for **b)**) with a statistical analysis based on fewer assumptions. Explain how you would perform such an analysis; specifically, state the analytical approach and its assumptions, and either carry out the proposed analysis from the information provided or describe in detail how you would perform the analysis in statistical software. In any case, make sure to describe the type of conclusion you will obtain from the analysis, e.g. in terms of the parameter(s) or feature(s) of the distribution(s) examined.
  - ii) (2 points) Due to the relatively small sample size, the study could be considered as a pilot study for a more extensive study to follow. Assume that the targeted margin

of error for 90% confidence intervals for such a subsequent study is (at most) 0.5 percentage points. Based on this information and a selected parameter of interest (reflecting the same purpose as in the pilot study), carry out a calculation to obtain a recommended sample size. If you are unable to calculate a recommended sample size, describe what information is missing for the calculation and/or describe in detail how you would use the statistical software to carry out such a calculation. Finally, with either the calculated or an assumed sample size (to be obtained as you have described) outline the resulting layout of the proposed study.

- iii) (2 points) One other important consideration was whether the grouping of rats in sets (as outlined in the description of the study) was an important feature to retain for future studies. First, discuss what advantages the researchers might have anticipated from grouping the rats in sets. Next, describe how you could use graphical or numerical methods to illustrate and/or assess the usefulness of the sets in the statistical analysis. Carry out as much of the proposed analysis as possible from the information provided, and draw conclusions.

Minitab listings (slightly edited):

```
MTB > Print 'set'-'raw-roasted'.
```

Data Display

Row	set	sex	digest_raw	digest_roast	raw-roasted
1	1	f	97.7	95.8	1.9
2	2	f	99.2	98.0	1.2
3	3	f	96.1	94.4	1.7
4	4	f	95.9	96.8	-0.9
5	5	m	97.2	97.1	0.1
6	6	m	99.5	96.2	3.3
7	7	m	98.6	96.5	2.1
8	8	m	97.2	95.0	2.2
9	9	m	96.6	95.0	1.6
10	10	m	95.9	96.7	-0.8

```
MTB > OneZ 'digest_raw' 'digest_roast' 'raw-roasted';
```

```
SUBC> Sigma 1.5;
```

```
SUBC> Test 0;
```

```
SUBC> Alternative 0.
```

```
One-Sample Z: digest_raw, digest_roast, raw-roasted
```

```
Test of mu = 0 vs <> 0
```

```
The assumed standard deviation = 1.5
```

Variable	N	Mean	StDev	SE Mean	95% CI	Z	P
digest_raw	10	97.390	1.337	0.474	(96.460, 98.320)	205.32	0.000
digest_roast	10	96.150	1.106	0.474	(95.220, 97.080)	202.70	0.000
raw-roasted	10	1.240	1.363	0.474	( 0.310, 2.170)	2.61	0.009

```
MTB > OneT 'digest_raw' 'digest_roast' 'raw-roasted';
SUBC> Test 0;
SUBC> Alternative 0.
One-Sample T: digest_raw, digest_roast, raw-roasted
```

Test of  $\mu = 0$  vs  $\neq 0$

Variable	N	Mean	StDev	SE Mean	95% CI	T	P
digest_raw	10	97.390	1.337	0.423	(96.434, 98.346)	230.34	0.000
digest_roast	10	96.150	1.106	0.350	(95.359, 96.941)	274.96	0.000
raw-roasted	10	1.240	1.363	0.431	( 0.265, 2.215)	2.88	0.018

```
MTB > TwoSample 'digest_raw' 'digest_roast';
SUBC> Test 0.0;
SUBC> Alternative 0;
SUBC> Pooled.
Two-Sample T-Test and CI: digest_raw, digest_roast
```

Two-sample T for digest\_raw vs digest\_roast

	N	Mean	StDev	SE Mean
digest_raw	10	97.39	1.34	0.42
digest_roast	10	96.15	1.11	0.35

Difference =  $\mu$  (digest\_raw) -  $\mu$  (digest\_roast)  
 Estimate for difference: 1.240  
 95% CI for difference: (0.087, 2.393)  
 T-Test of difference = 0 (vs  $\neq$ ): T-Value = 2.26 P-Value = 0.036 DF = 18  
 Both use Pooled StDev = 1.2269

```
MTB > TwoSample 'digest_raw' 'digest_roast';
SUBC> Test 0.0;
SUBC> Alternative 0.
Two-Sample T-Test and CI: digest_raw, digest_roast
```

Two-sample T for digest\_raw vs digest\_roast

	N	Mean	StDev	SE Mean
digest_raw	10	97.39	1.34	0.42
digest_roast	10	96.15	1.11	0.35

Difference =  $\mu$  (digest\_raw) -  $\mu$  (digest\_roast)  
 Estimate for difference: 1.240  
 95% CI for difference: (0.082, 2.398)  
 T-Test of difference = 0 (vs  $\neq$ ): T-Value = 2.26 P-Value = 0.037 DF = 17

*(continues on next page)*

```
MTB > TwoT 'digest_raw' 'sex';
SUBC> Test 0.0;
SUBC> Alternative 0.
Two-Sample T-Test and CI: digest_raw, sex
```

Two-sample T for digest\_raw

sex	N	Mean	StDev	SE Mean
f	4	97.22	1.54	0.77
m	6	97.50	1.32	0.54

Difference =  $\mu$  (f) -  $\mu$  (m)  
Estimate for difference: -0.275  
95% CI for difference: (-2.697, 2.147)  
T-Test of difference = 0 (vs <>): T-Value = -0.29 P-Value = 0.782 DF = 5

```
MTB > TwoT 'digest_roast' 'sex';
SUBC> Test 0.0;
SUBC> Alternative 0.
Two-Sample T-Test and CI: digest_roast, sex
```

Two-sample T for digest\_roast

sex	N	Mean	StDev	SE Mean
f	4	96.25	1.53	0.76
m	6	96.083	0.889	0.36

Difference =  $\mu$  (f) -  $\mu$  (m)  
Estimate for difference: 0.167  
95% CI for difference: (-2.180, 2.513)  
T-Test of difference = 0 (vs <>): T-Value = 0.20 P-Value = 0.853 DF = 4

```
MTB > TwoT 'raw-roasted' 'sex';
SUBC> Test 0.0;
SUBC> Alternative 0.
Two-Sample T-Test and CI: raw-roasted, sex
```

Two-sample T for raw-roasted

sex	N	Mean	StDev	SE Mean
f	4	0.98	1.28	0.64
m	6	1.42	1.50	0.61

Difference =  $\mu$  (f) -  $\mu$  (m)  
Estimate for difference: -0.442  
95% CI for difference: (-2.542, 1.659)  
T-Test of difference = 0 (vs <>): T-Value = -0.50 P-Value = 0.634 DF = 7

Stata listings:

```
. version 15  
. list , ab(12)
```

	set	sex	digest_raw	digest_roast	raw_roast
1.	1	f	97.7	95.8	1.9
2.	2	f	99.2	98	1.2
3.	3	f	96.1	94.4	1.7
4.	4	f	95.9	96.8	-.9
5.	5	m	97.2	97.1	.1
6.	6	m	99.5	96.2	3.3
7.	7	m	98.6	96.5	2.1
8.	8	m	97.2	95	2.2
9.	9	m	96.6	95	1.6
10.	10	m	95.9	96.7	-.8

```
. ztest digest_raw=0, sd(1.5)
```

One-sample z test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
digest~w	10	97.39	.4743416	1.5	96.46031 98.31969

mean = mean(digest\_raw) z = 205.3161  
Ho: mean = 0

Ha: mean < 0                      Ha: mean != 0                      Ha: mean > 0  
Pr(Z < z) = 1.0000                  Pr(|Z| > |z|) = 0.0000                  Pr(Z > z) = 0.0000

```
. ztest digest_roast=0, sd(1.5)
```

One-sample z test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
digest~t	10	96.15	.4743416	1.5	95.22031 97.07969

mean = mean(digest\_roast) z = 202.7020  
Ho: mean = 0

Ha: mean < 0                      Ha: mean != 0                      Ha: mean > 0  
Pr(Z < z) = 1.0000                  Pr(|Z| > |z|) = 0.0000                  Pr(Z > z) = 0.0000

```
. ztest raw_roast=0, sd(1.5)
```

One-sample z test

```

-----
Variable |      Obs      Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
raw_roast |       10      1.24   .4743416     1.5     .3103075     2.169693
-----
      mean = mean(raw_roast)                                z =    2.6141
Ho: mean = 0

      Ha: mean < 0                Ha: mean != 0                Ha: mean > 0
Pr(Z < z) = 0.9955          Pr(|Z| > |z|) = 0.0089          Pr(Z > z) = 0.0045

```

```

. ttest digest_raw=0
One-sample t test

```

```

-----
Variable |      Obs      Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
digest_raw |       10     97.39   .4228078     1.337036   96.43354     98.34646
-----
      mean = mean(digest_raw)                                t = 230.3410
Ho: mean = 0                                                degrees of freedom =      9

      Ha: mean < 0                Ha: mean != 0                Ha: mean > 0
Pr(T < t) = 1.0000          Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000

```

```

. ttest digest_roast=0
One-sample t test

```

```

-----
Variable |      Obs      Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
digest_roast |       10     96.15   .3496822     1.105792   95.35896     96.94104
-----
      mean = mean(digest_roast)                              t = 274.9639
Ho: mean = 0                                                degrees of freedom =      9

      Ha: mean < 0                Ha: mean != 0                Ha: mean > 0
Pr(T < t) = 1.0000          Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000

```

```

. ttest raw_roast=0
One-sample t test

```

```

-----
Variable |      Obs      Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
raw_roast |       10      1.24   .431071     1.363166   .2648496     2.21515
-----
      mean = mean(raw_roast)                                t =    2.8766
Ho: mean = 0                                                degrees of freedom =      9

```



```

      m |          6          97.5   .5403702   1.323631   96.11093   98.88906
-----+-----
combined |         10          97.39   .4228078   1.337036   96.43354   98.34646
-----+-----
      diff |          -.2750003   .9421378          -2.597441   2.047441
-----+-----
      diff = mean(f) - mean(m)                                t =  -0.2919
Ho: diff = 0                                Satterthwaite's degrees of freedom =  5.8228

      Ha: diff < 0                                Ha: diff != 0                                Ha: diff > 0
Pr(T < t) = 0.3902                                Pr(|T| > |t|) = 0.7805                                Pr(T > t) = 0.6098

```

```

. ttest digest_roast, by(sex) unequal
Two-sample t test with unequal variances

```

```

-----+-----
      Group |         Obs          Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
      f |          4          96.25   .7632166   1.526433   93.82111   98.6789
      m |          6          96.08333   .3627821   .8886311   95.15077   97.01589
-----+-----
combined |         10          96.15   .3496822   1.105792   95.35896   96.94104
-----+-----
      diff |          .1666698   .8450506          -2.10238   2.43572
-----+-----
      diff = mean(f) - mean(m)                                t =  0.1972
Ho: diff = 0                                Satterthwaite's degrees of freedom =  4.3748

      Ha: diff < 0                                Ha: diff != 0                                Ha: diff > 0
Pr(T < t) = 0.5738                                Pr(|T| > |t|) = 0.8525                                Pr(T > t) = 0.4262

```

```

. ttest raw_roast, by(sex) unequal
Two-sample t test with unequal variances

```

```

-----+-----
      Group |         Obs          Mean   Std. Err.   Std. Dev.   [95% Conf. Interval]
-----+-----
      f |          4           .975   .6420994   1.284199   -1.068447   3.018447
      m |          6          1.416667   .613958   1.503884   -.1615627   2.994896
-----+-----
combined |         10           1.24   .431071   1.363166   .2648496   2.21515
-----+-----
      diff |          -.4416666   .8883896          -2.523826   1.640492
-----+-----
      diff = mean(f) - mean(m)                                t =  -0.4972
Ho: diff = 0                                Satterthwaite's degrees of freedom =  7.32136

      Ha: diff < 0                                Ha: diff != 0                                Ha: diff > 0
Pr(T < t) = 0.3168                                Pr(|T| > |t|) = 0.6337                                Pr(T > t) = 0.6832

```