Supplementary exercise 1.111 of IPS7e

Also for this problem we will demonstrate both the calculations from the statistical table for the standard normal distribution, e.g. Table B of PSLS, and two ways of getting the desired values using Minitab. We denote by Z a random variable from $\mathrm{N}(0, 1)$, the standard normal distribution.

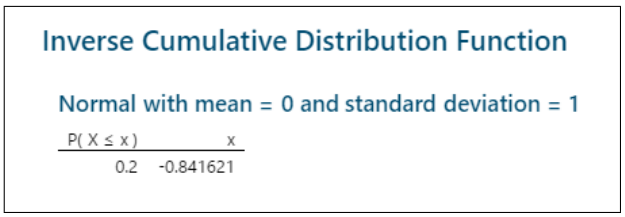
(a) In the normal distribution table, we need to look for z such that $P(Z < z) = 0.20$ (that is, z is the 20% percentile in the standard normal distribution). The closest approximation is $z = -0.84$ (closest with $P(Z < z) = 0.2005$).

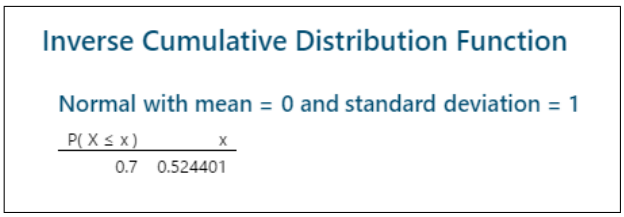
(b) Here we should look for z such that $P(Z > z) = 0.30$ (that is, z is the $(100 - 30)\% = 70\%$ percentile in the standard normal distribution). The closest approximation is $z = 0.52$ (with $P(Z < z) = 0.6985$).

Because the approximation for $z = 0.53$ is almost as good ($P(Z < 0.53) = 0.7019$), it seems obvious that values between 0.52 and 0.53 will be better approximations. It is valid to approximate the z-value by interpolation from the table. As 0.6985 and 0.7019 are almost equally far away from 0.70, the midpoint (that is, $z = (0.52 + 0.53)/2 = 0.525$) should be a good approximation, and the more exact calculation in statistical software confirms this. A more elaborate interpolation could be done, but if a more precise value is desired it seems more natural to use statistical software.

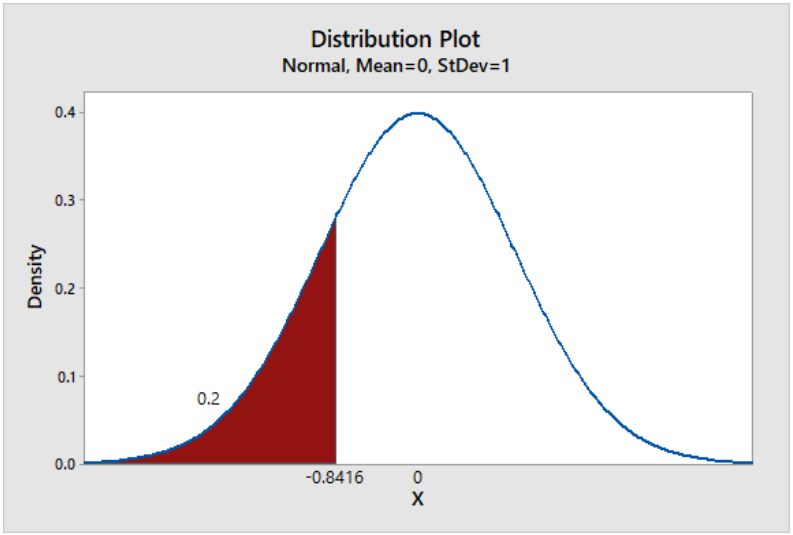
Next we demonstrate two ways in which Minitab can be used for the calculations. In the table-type menu, `Calc-Probability Distributions` menu, after having chosen the normal distribution, there is direct access to "Inverse cumulative probability", i.e., that reverse calculation from probability to z-value we are doing here. As with cumulative probability calculations, it's simpler to enter the desired value for which we want to compute the inverse cumulative probability as "Input constant". Because only inverse cumulative probabilities can be computed by this function, the probability entered needs to be a cumulative probability (a probability "to the left"). Minitab commands and output for the two calculations from above:

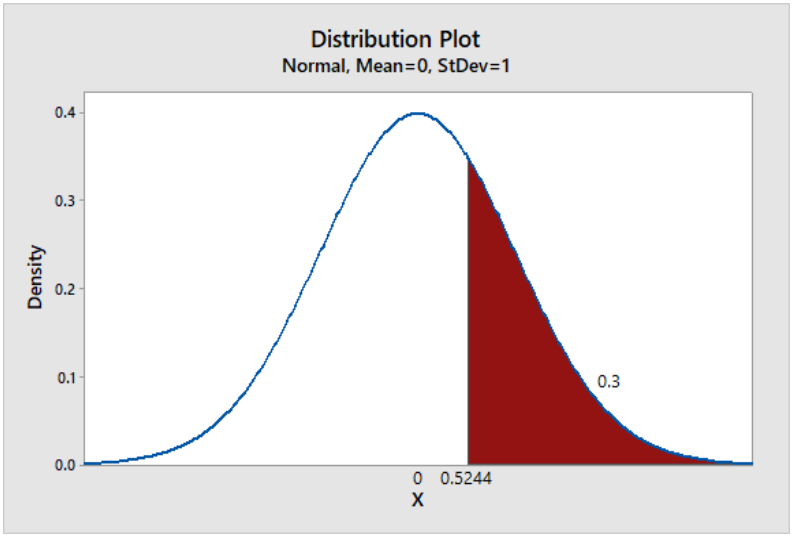
```
InvCDF 0.2;
 Normal 0.0 1.0.
InvCDF 0.7;
 Normal 0.0 1.0.
```





The second way to do these calculations in Minitab uses the graphical display in the menu obtained from `Graph-Probability Distribution Plot`. Select the plot type by "View Probability" and "Shared Area", after having selected the normal probability distribution of interest. After having chosen the relevant area, enter the probability of the area; note that when choosing the area type as "Right Tail" we should not convert the probability in (b) to a cumulative probability. The next page shows the corresponding graphical outputs.


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0.2
-0.8416
0
X


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0.3
0
0.5244
X