Exercise 27.22 of PSLS3e

Data: Counts of cereal leaf beetles trapped on boards (covered with a sticky material) of four different colors in farm fields.

Model: If we denote by X_{ij} the beetle count on the jth board of color i (where $i = 1, 2, 3, 4$ and $j = 1, \ldots, 6$), the statistical model is that the X_{ij}'s for each colour i constitute a simple random sample from a suitable distribution (alternatively, we could say that the 6 values for each color are i.i.d.). More discussion of model assumptions will follow below.

(a) For descriptive statistics, refer to the Minitab table. We could display the data by comparative dotplots (results not shown).

```
Describe 'beetles';
 By 'color'.
Dotplot ( 'beetles' ) * 'color'.
```

Descriptive Statistics: beetles

Statistics

Variable	color	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
beetles	Blue	6	0	14.83	2.18	5.34	7.00	10.00	15.00	20.25	21.00
	Green	6	0	31.17	2.57	6.31	20.00	26.75	32.00	37.00	37.00
	White	6	0	16.17	1.54	3.76	12.00	12.75	15.50	20.25	21.00
	Yellow	6	0	47.17	2.77	6.79	38.00	43.25	46.50	50.75	59.00

Comments:
The table of descriptive statistics shows the medians for the 4 groups. As with the means, the color lemon-yellow appears most attractive, followed by green, and finally white and blue seem least and about equally (un)attractive.

(b) For the ANOVA, we further assume that the X_{ij} follow normal distributions $\mathrm{N}(\mu_i, \sigma)$, and the hypotheses are:

$$\mathrm{H}_0: \ \mu_1 = \mu_2 = \mu_3 = \mu_4 \quad \text{(equal mean no. of insects trapped for the 4 colors),}$$
$$\mathrm{H}_a: \ \text{not } \mathrm{H}_0\text{, that is, some colors have higher mean no. of insects trapped than others.}$$

For the Kruskal-Wallis test, there is the option to additionally assume equal distribution shapes (the "Δ-assumption") for the four groups. With that assumption, the hypotheses can be expressed in terms of the medians η_i for each group,

$$\mathrm{H}_0: \ \eta_1 = \eta_2 = \eta_3 = \eta_4 \quad \text{(equal median no. of insects trapped for the 4 colors),}$$
$$\mathrm{H}_a: \ \text{not } \mathrm{H}_0\text{, that is, some colors have higher median no. of insects trapped than others.}$$

Without the "Δ-assumption", the Kruskal-Wallis test is for the hypothesis of equal distributions of beetle counts for the four colors, against the two-sided alternative that some differences between the distributions exist. With only six observations per group, it is quite impossible to validate the "Δ-assumption" from the data.

(c) In the PSLS notation, we have: $k = 4$; $n_1 = n_2 = n_3 = n_4 = 6$; $N = 24$.

(d) We use Minitab to carry out the calculations:

Kruskal-Wallis Test: beetles versus color

Descriptive Statistics

color	N	Median	Mean Rank	Z-Value
Blue	6	15.0	6.3	-2.47
Green	6	32.0	15.0	1.00
White	6	15.5	7.2	-2.13
Yellow	6	46.5	21.5	3.60
Overall	24		12.5	

Test

Null hypothesis H_0: All medians are equal
Alternative hypothesis H_1: At least one median is different

Method	DF	H-Value	P-Value
Not adjusted for ties	3	18.45	0.000
Adjusted for ties	3	18.51	0.000

Comments:
The Kruskal-Wallis test is strongly significant (P<0.0005), so we conclude that the 4 distributions are not equal. Also, if we were willing to assume equal distribution shapes, it may be concluded that the medians are different.

Additional questions: 1-way ANOVA analysis
(Note that the 1-way ANOVA analysis is also in Example 24.4 of PSLS, and corresponds to Supplementary exericse 12.23 for IIPS7e).

The table of descriptive statistics shows that the results for the four colors differ clearly in their means, while their standard deviations are pretty similar. The IPS/PSLS guideline for equal variances is met: $6.79/3.76 = 1.8$. There is some indication that higher standard deviations are associated with higher means, so one could try to transform the data by a log or square-root transformation. However, because we meet the guideline even with a fairly small sample size, and as the Minitab listing below shows the tests for equal variances (`Stat-ANOVA-Test for Equal Variances`) to be nowhere near significant, it does not seem necessary to transform the data.

```
VarTest 'beetles' 'color';
Confidence 95.0;
GInterval;
 NoDefault;
 TMethod;
 TBonferroni;
 TTest.
```

Test for Equal Variances: beetles versus color

Method

Null hypothesis	All variances are equal
Alternative hypothesis	At least one variance is different
Significance level	α = 0.05

95% Bonferroni Confidence Intervals for Standard Deviations

color	N	StDev	CI
Blue	6	5.34478	(2.30585, 21.2240)
Green	6	6.30608	(1.64163, 41.4995)
White	6	3.76386	(1.63904, 14.8073)
Yellow	6	6.79461	(1.77430, 44.5760)

Individual confidence level = 98.75%

Tests

Method	Test Statistic	P-Value
Multiple comparisons	—	0.732
Levene	0.11	0.955

Test for Equal Variances: beetles vs color

We could also test for normality within each of the four groups (all non-significant), but because of the small sample sizes we should not attribute too much importance to the results of such normality tests. The is a situation where model checking using residuals (Session 12) will be more effective.

One-way ANOVA: beetles versus color

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
color	4	Blue, Green, White, Yellow

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
color	3	4134.0	1378.00	42.84	0.000
Error	20	643.3	32.17		
Total	23	4777.3			

Model Summary

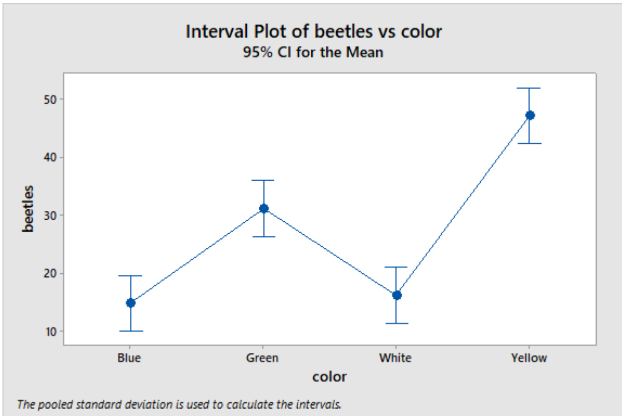
S	R-sq	R-sq(adj)	R-sq(pred)
5.67157	86.53%	84.51%	80.61%

Means

color	N	Mean	StDev	95% CI
Blue	6	14.83	5.34	(10.00, 19.66)
Green	6	31.17	6.31	(26.34, 36.00)
White	6	16.17	3.76	(11.34, 21.00)
Yellow	6	47.17	6.79	(42.34, 52.00)

Pooled StDev = 5.67157

Interval Plot of beetles vs color



Comments:
The F-statistic is strongly significant ($P < 0.0005$ from the Minitab listing). Regardless of any (minor) concerns with model assumptions we can confidently conclude that the colors do not attract the same number of insects. The figure indicates that yellow seems to be the strongest attractor, followed by green, whereas there is no obvious difference between blue and white. We supplement the figure with two LSD-values, an uncorrected (informal) and a Bonferroni-corrected ($4 \cdot 3/2 = 6$ comparisons) value:

$$\begin{aligned} \text{LSD}(0.95) &= t^* s\sqrt{2/6} = 2.086 \cdot 5.67157 \cdot \sqrt{2/6} = 6.8, \\ \text{LSD}(1-0.05/6) &= \text{LSD}(0.9917) = 2.927 \cdot 5.67157 \cdot \sqrt{2/6} = 9.6, \end{aligned}$$

where the second t^*-value was determined in Minitab from the $t(20)$-distribution for a tail probability of $p = 0.025/6 = 0.00417$. It is seen that both LSD-values lead to the same conclusion, namely that those colors visibly separated in the figure are also significantly different.