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## PRACTICAL INFORMATION

### Today's session:

- mixed models for **discrete data**<sup>1</sup>, with focus on random effects logistic regression,
  - \* illustrated by two-level pig dataset (pig\_adg) previously explored,
- “**alternative methods**” — to the linear and logistic **mixed models**:<sup>2</sup>
  - \* perhaps more relevant for discrete than continuous data<sup>3</sup>,
  - \* brief review of several methods, contrasting their properties,
  - \* main focus on **generalized estimating equations** (GEE), but really just an introduction to a large topic,
  - \* illustrations and comparisons of methods by familiar **2-level datasets**: simulated datasets (Chapter 20) and pig\_adg data.

### News/Schedule:

- home assignment 4 and project proposal due on Monday (March 17),
- two VER problems posted, to work with for Monday's lab session.

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<sup>1</sup> Corresponding to Sections 22.1-4 of the VER2/MER textbook (with some skipings).

<sup>2</sup> VER2/MER textbook coverage: alternative methods: Section 20.5, GEE: Section 23.5.

<sup>3</sup> Linear mixed models remain the most popular choice when its assumptions can be met.

## MIXED MODELS FOR DISCRETE DATA

### Synthesis:

discrete mixed models — usually called **generalized linear mixed models** (GLMMs) — extend and combine linear mixed models (LMMs) and generalized linear models (GLMs),

- **intuitive and flexible modelling** of clustering by means of **random effects**,
- but interpretation of models are less straightforward than in LMMs and GLMs,
  - \* fixed effects come in **two versions**: cluster-specific and population-averaged,
  - \* **variance components** more difficult to estimate meaningfully (and maybe variances are less meaningful for discrete data anyway),
- and **statistical analysis of GLMMs** is not as simple as we would want/are used to: many methods exist<sup>4</sup> and none has proven generally superior or always useful.

### Contents:

- very brief introduction to random effects logistic regression model,
- overview of statistical inference (but limited details),
- brief Stata demonstration (**note**: models not available in Minitab).

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<sup>4</sup> Referring to methods for estimation and inference; alternative modelling approaches exist as well (later in lecture).

## RANDOM EFFECTS IN LOGISTIC REGRESSION

Logistic regression: the **linear modelling of  $x$ 's** is at logit scale.

**Idea:** put random effects also at logit scale.

**Somatic cell count example** (conceptual continuation):

- Let  $Y_{ij}$  = binary disease status wrt. clinical mastitis for cow  $i$  in herd  $j$ ,
- let  $p_{ij} = \Pr(Y_{ij} = 1)$  (disease probability),
- model with **random herd effects** ( $u_j$ ):

$$\text{logit}(p_{ij}) = \beta_0 + \beta_1 x_{1ij} + \dots + \beta_k x_{kij} + u_j, \quad (1)$$

where  $u_j \sim N(0, \sigma_h^2)$ , and  $\sigma_h$  is the dispersion (standard deviation) among herds in logit(disease probabilities):

- \*  **$\sigma_h = 0$** : no disease clustering in herds (after accounting for fixed effects),
- \*  **$\sigma_h$  large**: strong disease clustering within herds (obs. in a herd are very alike), and large variation between herds (obs. are very different between herds).
- \* value of  $\sigma_h$  has no simple interpretation for  $p$ 's.

**Note:** Equation (1) is **conditional** on the  $u_j$ 's and gives the probability of disease for fixed  $u_j$ 's<sup>5</sup>; animals within a herd are also assumed conditionally independent.

<sup>5</sup> To compute  $p_{ij}$  when the herd random effect is unknown (a **marginal** probability) is difficult and can only be done numerically; Stata's margins command will do this after some model estimations, see 10bL–22.

## RANDOM EFFECTS IN GENERALIZED LINEAR MODELS

We build a GLMM from a GLM in the same way as with the random effects logistic regression:

- put the **random effects at the transformed scale** (determined by the link function), for example,

- \* **binary regression** with link function  $g$  (logit or alternatives):

$$g(\mu_i) = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + u_{\text{herd}(i)},$$

- \* **Poisson regression** with log link:

$$\ln(\lambda_i) = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + u_{\text{herd}(i)},$$

- \* **general GLMM formulation:**

$$g(\mu) = \eta = X'\beta + Zu, \tag{2}$$

where  $Z$  is the design matrix for the random effects,

- **more than one** set of random effects is possible, e.g. for a 3-level hierarchy,
- the notation  $Zu$  is just short for all random effects in the model,

- models with a **cluster-specific** (as opposed to population-averaged) **interpretation** of the fixed effects parameters, see 10bL–7,
- **no simple variance decomposition** (as we had for linear mixed models); see 10bL–8 for logistic model, and textbook Chapter 22 for Poisson model.

## PIG DATA EXAMPLE

We revisit the `pig_adg` dataset (Model-building exercise), and occurrence of enzootic pneumonia and its dependence on atrophic rhinitis (dichotomized at  $ar > 1$ , `ar_g1`).

Data structure:

Level	Number	Mean	Range
Farm	15	—	—
Pig	341	22.7	14–28

$2 \times 2$  table analysis:

simple OR = 1.909

$\ln(\text{OR}) = 0.647$

ar_g1	pn		Total
	1	0	
1	109	66	175
0	77	89	166
Total	186	155	341

Simple logistic regression for pn:

$$p_{ij} = \Pr(\text{pn}_{ij} = 1), \quad \text{for pig } i \text{ in herd } j,$$

$$\text{logit}(p_{ij}) = \beta_0 + \beta_1 \text{ar\_g1}_{ij},$$

gives  $\hat{\beta}_0 = -0.145$  (SE = 0.258) and  $\hat{\beta}_1 = 0.647$  (SE = 0.220).

Expand to a **random effects** logistic regression to account for herds:

$$\text{logit}(p_{ij}) = \beta_0 + \beta_1 \text{ar\_g1}_{ij} + u_j,$$

where  $u_j$ 's are the random herd effects, assumed  $\sim N(0, \sigma_h^2)$ .

- $\hat{\beta}_1 = 0.437$  (SE = 0.258) — **much smaller** than above, probably due to confounding by herds, and also **much weaker** ( $P = 0.091$  compared to  $P = 0.003$ ),
- $\hat{\sigma}_h^2 = 0.877$  (SE = 0.433) — some herd clustering and clearly significant,
- $\hat{\beta}_0 = 0.020$  (SE = 0.301) — larger SE because now a herd-level parameter.

## REUNION ISLAND DATA EXAMPLE

**Binary outcome** = conception at first service (yes/no):

- observed in each lactation of the participating cows,
- 3-level data structure (disregarding again the 5 regions at the top level):  
3027 lactations — 1575 cows — 50 herds,
- overall conception rate:  $1302/3027 = 0.43$ ,
- **fixed effects**: heifer, ai and log calving to first service interval (lncfs).

**Random effects logistic regression**:

$$\text{logit}(p_{ijk}) = \beta_0 + \beta_1 \text{heifer}_{ijk} + \beta_2 \text{ai}_{ijk} + \beta_3 \text{lncfs}_{ijk} + v_k + u_{jk}, \quad \text{where}$$

- $p_i$  = prob. of conception in lactation  $i$  of cow  $j$  in herd  $k$ ,
- $v_k \sim N(0, \sigma_h^2)$  and  $u_{jk} \sim N(0, \sigma_c^2)$ .

**Parameter estimates** (from Stata's melogit command):

intercept :  $\hat{\beta}_0 = -1.515$  (SE=.446); heifer coef. :  $\hat{\beta}_1 = -0.062$  (SE=.097),

ai coef. :  $\hat{\beta}_2 = -1.023$  (SE=.131); lncfs coef. :  $\hat{\beta}_3 = 0.497$  (SE=.102),

herd variance :  $\hat{\sigma}_h^2 = 0.073$  (SE=.036); cow variance :  $\hat{\sigma}_c^2 = 0.280$  (SE=.122).

— very small random effects...

## CLUSTER-SPECIFIC VERSUS POPULATION-AVERAGED INTERPRETATIONS

**Odds-ratio** (OR) computed as usual:  $OR = \exp(\hat{\beta}_r)$ : e.g.,  $\exp(0.44) = 1.55$  for ar\_g1.

This OR has a **cluster-specific** interpretation:

- it refers to the difference in risk when comparing two pigs **within the same farm**, with/without the ar\_g1 condition,
- no assumptions are made about the actual farm (its random effect),
- “**cluster-specific**”  $\sim$  for a specific farm; also (traditionally) denoted “**subject-specific**”<sup>6</sup>, but here it does not refer to individuals!,
- the odds-ratio can be thought of as the value relevant to the farmer.

Alternative<sup>7</sup>: a **population-averaged** (or **marginal**) interpretation (directly from some statistical procedures, or by the approximation formula:  $\beta_{PA} \approx \beta_{CS} / \sqrt{1 + 0.346 \sigma_h^2}$ ):

- it refers to the difference in risk when comparing two pigs, with/without the ar\_g1 condition, **from the entire population** of pigs (not from the same farm),
- “**population-averaged**”  $\sim$  across the population (of farms),
- the odds-ratio can be thought of as the value relevant to the association of farmers (or the slaughterhouse).

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<sup>6</sup> The term “subject-specific” stems from repeated measures data, where individuals can be thought of as clusters; VHM 802 Session 12.

<sup>7</sup> Cluster-specific and population-averaged estimates agree generally in linear mixed models, and also in GLMMs when clustering is absent.

## VARIANCE COMPONENTS IN GLMMs

Interpretation of **random effect variances**:

- strictly speaking, as the **variation in logit( $p_i$ )**, or  $g(\mu_i)$ , between farms (clusters) after having accounted for fixed effects,
- **no simple variance decomposition** in GLMMs, because the model has
  - \* no error term ( $\varepsilon$ ) and hence no  $\sigma^2$  in the model equation,<sup>8</sup>
  - \* the total variance depending on the mean (and therefore **not constant!**).

**Approximate method** for random effects logistic regression<sup>9</sup>:

- put  $\sigma^2 = \pi^2/3 = 3.29$ , and proceed as for linear mixed models,
- gives **approximate** variances/correlations (albeit constant!) for the binary outcome  $Y$ ,
- gives **exact** variances/correlations for a continuous **latent** (unobserved) **variable  $\ell$**  associated with every cow,  $\ell \sim$  “degree of sickness” (on some scale),
  - \* binary outcome  $Y$  corresponds to thresholding of underlying  $\ell$  at **cut-off value  $t$** :

$$\begin{array}{c}
 0 \qquad \qquad \qquad 1 \qquad \qquad \qquad Y \\
 \hline
 \qquad \qquad \qquad t \qquad \qquad \qquad \ell
 \end{array}
 \quad \text{formally, } Y = \begin{cases} 1, & \ell \geq t, \\ 0, & \ell < t, \end{cases}$$

<sup>8</sup> The variation is on two different scales: observation scale and logit scale ( $\sigma_h^2$ ).

<sup>9</sup> Reference for this/other methods: Vigre et al. (2004), *Preventive Veterinary Medicine* 63, 9–28.

## STATISTICAL INFERENCE FOR GLMMs

**Estimation** is not straightforward<sup>10</sup>:

- different methods exist; our focus here is on **maximum likelihood** estimation in Stata,<sup>11</sup>
  - \* considered among best methods whenever feasible (Bayesian estimation also good, but beyond our current scope),
  - \* numerically difficult for large datasets/models,
  - \* cautious modelling recommended, for example by comparison with other approaches (e.g. GEE, later slides).

**Tests and confidence intervals** (approximate, even if model assumptions are met)

- **fixed effects**: Wald test and CI (the usual way), likelihood-ratio test **preferable/necessary** in the presence of estimation problems (e.g. extreme fixed effects estimates),
- **random effects**: Wald test and CI very crude, preferred test is likelihood-ratio test (if any testing is needed...),
- **model checking**: involves multiple levels as in linear mixed models,
  - \* **lowest level** suffers from same limitations as in ordinary logistic regression<sup>12</sup>
  - \* **less statistics and procedures** available, and options strongly software dependent.

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<sup>10</sup> The main problem is that the likelihood function is difficult to compute, and can only be approximated.

<sup>11</sup> ML-estimation in Stata for GLMMs is superior to all other general statistics packages; in other software, one should check what approximation is used; some are of so poor quality that  $\log L$  values are pretty useless for inference.

<sup>12</sup> With the additional random effects, covariate patterns with sensible replication are even harder to get by.

## OVERVIEW: DEALING WITH CLUSTERING

### Detection of clustering:

- primarily through **understanding of data structure**,
- decisions about accounting for clustering should **not be based on statistical testing!**<sup>13</sup>

### Approaches to model clustering (in the course):

- mixed (random effects) models — covered already,

### Approaches to account for clustering (in the course):

- fixed effects,
- stratification, for binary data (Mantel-Haenszel procedure),
- robust standard errors,
- **generalized estimating equations** (GEE).

### Choice of approach should take into account:

- ease of use (but easy does not mean valid...),
- the assumptions required,
- (degree of) interest in clustering per se.

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<sup>13</sup> The reason is that even small (possibly statistically non-significant) clustering can have substantial impact (as discussed in terms of variance inflation), so we prefer to account for clustering whenever its presence makes biological sense.

## FIXED EFFECTS MODELLING

**Fixed effects modelling:** enter the herds (clusters) into the model as a categorical variable, to estimate a separate parameter for each herd (but one).

**Advantages and disadvantages** of fixed effects modelling:

- + generally very easy to carry out (e.g., a 2-level model will have no random effects),
- + avoids distributional assumption about herd effects,
- + avoids taking the herds as representative for a population (which might be inappropriate if the number of herds is small),
- +/- estimates are **cluster-specific** and specific to actual herds in the study (cannot be generalized to a population),
- **does not allow for herd-level predictors**,
- does not give an estimate of the variance between herds,
- may lead to biased estimates for other fixed effects when the number of herds is large, in particular for non-normal models.

**Results** for simulated datasets (cow-level predictor only):<sup>14</sup>

- o **linear model:**  $\hat{\beta} = 4.968 (.149)$  – close to mixed model,
- o **logistic model:**  $\hat{\beta} = 0.704 (.046)$  – close to mixed model.

<sup>14</sup> Estimates for the intercept  $\sim$  reference herd  $\Rightarrow$  not comparable to mixed models or models without herd effects.

## STRATIFICATION (BINARY DATA)

**Stratification** = Mantel-Haenszel procedures using herds (clusters) as strata:

- combined odds-ratio (OR) across herds (binary within-herd predictor; binary outcome),
- test for association in two-way table (categorical within-herd predictor; categorical outcome), adjusted for herds.

**Advantages and disadvantages** of stratification:

- + easy to carry out,
- + avoids any assumptions about the herds,
- +/- cluster-specific estimates (OR),
  - restricted scope and limited analysis,
  - no insight into the type or magnitude of clustering.

**Results** for simulated dataset (cow-level predictor only):

- o Mantel-Haenszel  $OR = 2.009 \Rightarrow \hat{\beta} = \ln(2.009) = 0.698$  – close to mixed model,
- o estimated  $SE = .046$ , also close to mixed model (computed by backtransforming OR and its confidence interval bounds to logit scale<sup>15</sup>).

<sup>15</sup> The CI for the M-H OR was constructed on logit scale, where estimates follow a normal distribution to a good approximation, and transformed to odds-ratio scale.

## ROBUST STANDARD ERRORS

### Background:

usual (model-based)  
statistical methods:

data  $\mapsto$  model  $\mapsto$   $\begin{cases} \text{estimates, test statistics} \\ \text{standard errors, } P\text{-values} \end{cases}$

- o statistical models are not always (never) true!
- o robust methods are designed to be **less sensitive** to model deviations.

### Robust variance estimation (Huber-White, “sandwich”):

- o base SEs on properties of the estimation method valid for a wider class of models than assumed,
- o method can be targeted to deal with clustering, where the assumption of within-group independence is critical (only this version is of interest here).

### Advantages and disadvantages of robust SE method:

- + simple to use and general method (available in Stata for wide range of models),
- +/- robust SEs have a different interpretation than usual SEs,
- +/- **does not affect the estimate**, only its standard error,
- gives no insight into the type or magnitude of clustering.

**Results:** fairly close to mixed model SEs (except: linear model, herd-level  $\mathbf{X}$ ).

## GENERALIZED ESTIMATING EQUATIONS (GEE)

Initial remarks about GEE for GLMs<sup>16</sup>:

- an estimation procedure (set of equations from which estimates are constructed iteratively) rather than a model,
- **partially specified model** involving only assumptions about the **marginal**<sup>17</sup> means and variances,
- gives **population-averaged** (or marginal) estimates,
- **no herd effects**, hence no assumed distribution for them,
- **no likelihood function** or likelihood-based inference.

Original form of GEE for GLMs:

- framework of longitudinal data (repeated measures),
- algorithm estimates a **working correlation matrix** for the correlation of observations within clusters (herds, subjects):
  - \* a setting in the algorithm, not a model assumption,
  - \* **hierarchical data**: use exchangeable type ( $\sim$  equal correlations  $\rightarrow$  next slide),
- invented in the 1980s and very much used since.

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<sup>16</sup> Recall that generalized linear models (GLMs) constitute a general class of models including logistic and Poisson regression, specified by a distribution (family) and a link function.

<sup>17</sup> “Marginal” refers to the mean across the population of clusters (herds).

## WORKING CORRELATION MATRICES

For a series  $Y = (Y_1, \dots, Y_n)$  of observations on a subject, the **correlation matrix**  $\text{Corr}(Y)$  is the  $n \times n$ -matrix of all pairs of correlations  $\sim Y$  as a multivariate outcome.

**Examples of working correlation matrices**<sup>18</sup> commonly used with GEE (shown for 4 observations per subject):

- **independence:**  
 $\sim$  independent or uncorrelated obs.,  
 or no assumptions about  $\text{Corr}(Y)$

$$\text{Corr}(Y_i, Y_j) = 0 \quad \text{for } i \neq j,$$

- **exchangeable:**  
 $\sim$  hierarchical data structure,  
 with  $\rho = \text{ICC}$  (also compound symmetry)

$$\text{Corr}(Y) = \begin{pmatrix} 1 & & & \\ \rho & 1 & & \\ \rho & \rho & 1 & \\ \rho & \rho & \rho & 1 \end{pmatrix},$$

- **autoregressive** ar(1):  
 $\sim$  first order autoregressive,  
 for repeated measures, has  
 decaying correlation with time

$$\text{Corr}(Y) = \begin{pmatrix} 1 & & & \\ \rho & 1 & & \\ \rho^2 & \rho & 1 & \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix},$$

- **Toeplitz:**  
 $\sim$  “stationary” ( $\rho$  depends on “distance” only)

$$\text{Corr}(Y) = \begin{pmatrix} 1 & & & \\ \rho_1 & 1 & & \\ \rho_2 & \rho_1 & 1 & \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{pmatrix}.$$

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<sup>18</sup> As the matrices are symmetric, for clarity the values above the diagonal are left blank.

## GEE INFERENCE AND VARIANTS

### GEE settings:

- strongly recommended to use **robust standard errors** instead of model-based ones (**caution**: not the Stata default),
- **note**: GEE with independent correlation matrix  $\sim$  robust variance estimation!,
- **hypothesis testing**: Wald tests mostly used (other tests exist),
- parameter estimates and SEs are **asymptotically unbiased** (when  $\#$  clusters is large) under weak conditions,<sup>19</sup>
- **performance in finite samples**: standard guideline on sample size is that at least 30 clusters is needed to avoid biases,

### Different versions of GEE for GLMs:

- differ by algorithms and settings for the part of the algorithm dealing with correlation structure — one important example: **alternating logistic regression** (ALR),
  - \* GEE-type procedure based on odds-ratios rather than working correlations (arguably more appropriate for binary data),
  - \* allows for either 2 or 3 hierarchical levels,
  - \* implemented in SAS and R, but not Stata.

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<sup>19</sup> Importantly, **misspecification** of the working correlation matrix **does not invalidate** the estimates, but may lead to some loss of efficiency.

## CHOICE OF CORRELATION “MODEL”

How to choose the “best” (working) correlation structure for GEE?

- in principle<sup>20</sup>, the QIC statistic (an analog of the AIC) can be compared between different correlation structures, and the one with smaller QIC value is preferable,
- **guidelines** (from Hardin and Hilbe (2003), *Generalized Estimating Equations*, 1st ed., p. 141–42):
  - \* for small cluster size and complete data, use unstructured,
  - \* for repeated measures over time, use a structure with time dependence, e.g. ar(1),
  - \* for 2-level hierarchical structure, use exchangeable,
  - \* for small number of clusters, independence structure may be best,
  - \* if more than one structure meets the guidelines, use the QIC statistic to choose between them,
  - \* unstructured correlations may be used for exploratory analysis.
- another **recommendation** (Hardin & Hilbe): **alternating logistic regression** is preferable to ordinary GEE for logistic regression if “the focus of the analysis does include the association parameters” (i.e., association within clusters).

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<sup>20</sup> My (Henrik’s) experience with the QIC statistic has been mixed, and I don’t feel comfortable with using QIC to choose the correlation structure.

## SUMMARY FOR GEE

**Advantages and disadvantages** of (classical) GEE method:

- + no assumptions about herd effects,
- + good/robust theoretical properties (with robust SE),
- + computationally feasible for large data sets,
- + possible/necessary to use knowledge of clustering,
- +/- **population-averaged** instead of cluster-specific estimates,<sup>21</sup>
  - less flexible with respect to multiple levels,
  - no direct modelling of correlation structure.
  - statistical choice of “working correlation structure” less clear,
  - choice between different GEE variants remains “subjective”.

**Results** (exchangeable correlation structure):

very close to linear mixed model (continuous data), but population-averaged estimates for binary data: 0.559 (.177) and 0.569 (.042) for herd- and cow-level  $X$ .

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<sup>21</sup> With identity link, e.g. linear models, there is no distinction between PA and CS estimates  
⇒ GEE is perfectly valid for normally distributed outcomes, but often linear mixed models are preferred, due to their more clear-cut inference and larger flexibility.

## SUMMARY OF ANALYSES FOR SIMULATED DATASETS

Collected **results** for the 4 datasets (two outcomes, two predictor levels):

Estimate (SE)	Continuous outcome (true = 5)		Binary outcome (true = 0.693) <sup>a</sup>	
Model/approach	herd-level $X$	cow-level $X$	herd-level $X$	cow-level $X$
unadjusted	3.557 (.200)	4.982 (.199)	0.529 (.042)	0.586 (.042)
mixed	3.796 (1.50)	4.968 (.149)	0.620 (.204)	0.697 (.046)
fixed effects	n/a	4.968 (.149)	n/a	0.704 (.046)
stratification	n/a	not discussed	n/a	0.698 (.046)
robust SE	3.557 (1.71)	4.982 (.142)	0.529 (.211)	0.586 (.044)
GEE (exch. corr)	3.797 (1.49)	4.968 (.141)	0.559 (.177)	0.569 (.042)

<sup>a</sup> cluster-specific true value, population-averaged equivalent  $\approx 0.597$   
 (computed as  $0.693 / \sqrt{1 + 0.346 \cdot 1} = 0.597$ )

### Extra notes:

- **cow-level predictor**: due to large sample size for  $X$ , all estimates are close to their (respective) true values,
- **herd-level predictor**: with an effective sample size of only 50 replicates per group, no close agreement with the true values can be expected.

## SUMMARY OF ANALYSES FOR PIG DATA

Estimates and SE (on logit scale):

Model	effect of ar_g1		intercept	
	Coef.	SE	Coef.	SE
ordinary LR	0.647	0.220	-0.145	0.156
fixed effects LR	0.365	0.268	n/a	
Mantel-Haenszel	0.346	0.261	n/a	
robust variance	0.647	0.267	-0.145	0.279
random effects LR	0.437	0.258	0.020	0.301
GEE (exch corr.)	0.354	0.215	0.018	0.271

Conclusion:

- estimate by **random effects** model (GLMM) somewhat larger than for GEE; more than explained by being a cluster-specific estimate, as seen from (10bL-7)

$$0.437 / \sqrt{1 + 0.346 \cdot 0.877} = 0.383,$$

but not critically off considering the SE,

- Mantel-Haenszel and fixed effects estimates should be closer to GLMM than GEE estimates; some herd-level confounding (for ar\_g1) appears to exist in the data,
- ordinary LR and robust SE considerably off, probably due to herd confounding.

## MEANS/MARGINS FOR (LINEAR) MIXED MODELS

**Basic fact:** decisions are **required** (or implicit) on how to deal with the random effects.

**Simplest situation:** predictions on same scale as the random effects (say  $u$ ):

- setting  $u = 0$  corresponds to predictions that are **means** in the random effects distribution,<sup>22</sup>
  - \* similar to setting  $\varepsilon = 0$  for prediction in linear models,
  - \* assumes prediction is for new “cluster” from population, as opposed to cluster(s) in the dataset,<sup>23</sup>
  - \* gives largest uncertainty (SE) for predictions because nothing is known about random effects from the data,
  - \* usual software default, e.g. in Stata’s margins command.

**More complex situation:** (non-linear) transformation of means, e.g. if outcome was transformed for mixed model analysis:

- similar to linear models, where (back)transforming means gives medians, not means,
- for  $u = 0$ , interpret transformed values as medians,
- exact means can be obtained analytically (using formulae) for some transformations and generally by simulation; in practice, the median interpretation is often sufficient and satisfactory.

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<sup>22</sup> With the usual assumption that random effects have mean 0, e.g.  $u \sim N(0, \sigma_h^2)$ .

<sup>23</sup> Sometimes termed referred to as “broad” and “narrow” inference spaces, respectively; Littell et al. (2006), *SAS for Mixed Models*, 2nd ed., SAS Publishing.

## MEANS/MARGINS FOR GLMMs AND GEE

**Main issue:** distinction between CS (cluster-specific) and PA (population-averaged) interpretations,

- PA predictions usually desired for broad<sup>23</sup> inference space,
  - \* directly with GEE (e.g. Stata's margins command),
  - \* caution needed (see below) for GLMMs,
- CS predictions may be of interest for specific clusters (included in the data), e.g. farms or schools — only available for GLMMs (technical: VER2/MER Ex. 22.10).

**Common software limitations** (Stata version  $\leq 13$ , SAS, R): no predictions on original scale in GLMMs,<sup>24</sup>

- can work around these by manual backtransformation, but it will not give proper PA predictions: **medians** instead of means (as discussed on previous page), and thus not truly population-averaged,
- analytical adjustment to achieve PA estimates may be possible, e.g. in logistic regression using the formula on slide 10bL–7.

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<sup>24</sup> In Stata 14+, only the `mefrlogit` and `mefrpoisson` commands do not allow estimation on original scale.