

Lecture 2b: Linear Regression Diagnostics

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- Exercises - Monday Lab
 - ★ Linear regression exercise 2
 - ★ Linear regression exercise 3

Example: WPC model (daisy2red.dta)

- VER example 14.12

- ★ outcome: wpc (interval from waiting period to conception)

- ★ predictors: parity, twin, dyst, rp, vag_disch, herd_size and herd_size2, calving_date (in months)

- ➔ also interactions: rp*vag_disch

- final (candidate) model

```
. reg wpc hs100_ctr hs100_ctr_sq parity1 aut_calv twin dyst rp##vag_disch, vsquish
```

Source	SS	df	MS	Number of obs =	1574
Model	296062.694	9	32895.8549	F(9, 1564) =	13.22
Residual	3892027.86	1564	2488.50886	Prob > F =	0.0000
-----				R-squared =	0.0707
Total	4188090.56	1573	2662.48605	Adj R-squared =	0.0653
-----				Root MSE =	49.885

wpc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
hs100_ctr	19.85708	2.163397	9.18	0.000	15.61361 24.10054
hs100_ctr_sq	11.13827	3.111145	3.58	0.000	5.035817 17.24073
parity1	1.13721	.8583103	1.32	0.185	-.5463501 2.82077
aut_calv	-8.263839	2.537751	-3.26	0.001	-13.24159 -3.286086
twin	20.68314	9.845165	2.10	0.036	1.37203 39.99425
dyst	11.70041	5.462576	2.14	0.032	.985666 22.41516
rp					
yes	5.98687	4.811976	1.24	0.214	-3.451734 15.42547
vag_disch					
yes	1.228196	7.161395	0.17	0.864	-12.81875 15.27514
rp#vag_disch					
yes#yes	22.85194	12.51605	1.83	0.068	-1.698056 47.40194
_cons	64.33029	2.634114	24.42	0.000	59.16352 69.49705

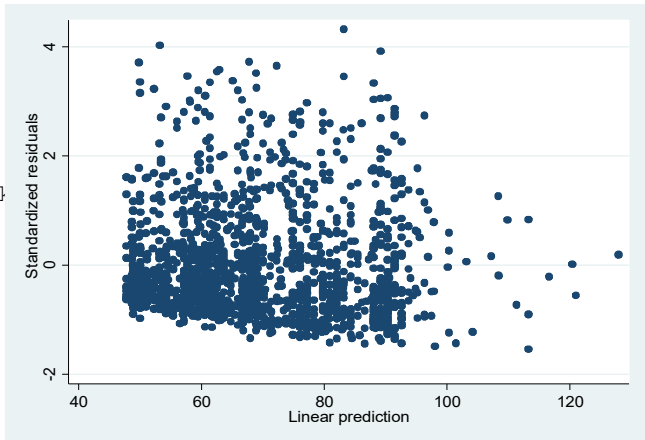
Evaluating major assumptions

● homoscedasticity

```
. hettest
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
Ho: Constant variance
Variables: fitted values of wpc

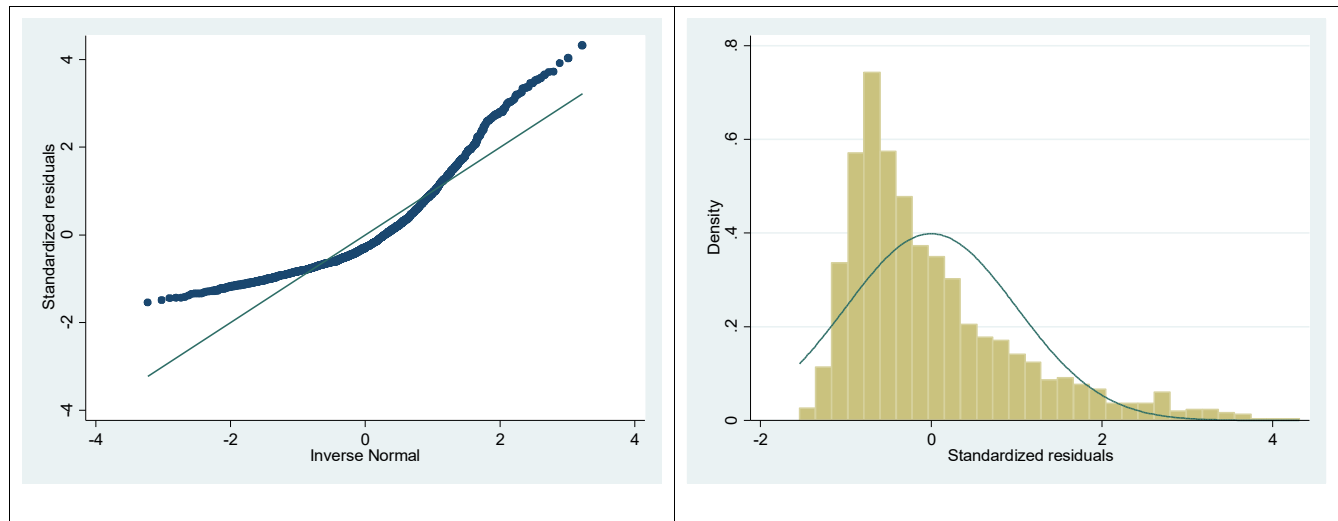
chi2(1)      =    20.58
Prob > chi2  =    0.0000

. imtest
Cameron & Trivedi's decomposition of IM-test
```



Source	chi2	df	p
Heteroskedasticity	74.11	44	0.0030
Skewness	143.84	9	0.0000
Kurtosis	33.27	1	0.0000
Total	251.22	54	0.0000

● normality



```
. swilk stdres
Shapiro-Wilk W test for normal data
```

Variable	Obs	W	V	z	Prob>z
stdres	1574	0.87871	115.660	11.977	0.00000

Transformation of Y (L1a - p12-15)

- Box-cox
- interpretation

Re-assessing assumptions

- Log-transformed model

```
. boxcox wpc aut_calv hsize hsize2 parity1 twin dyst rp vag_disch  
...output omitted...
```

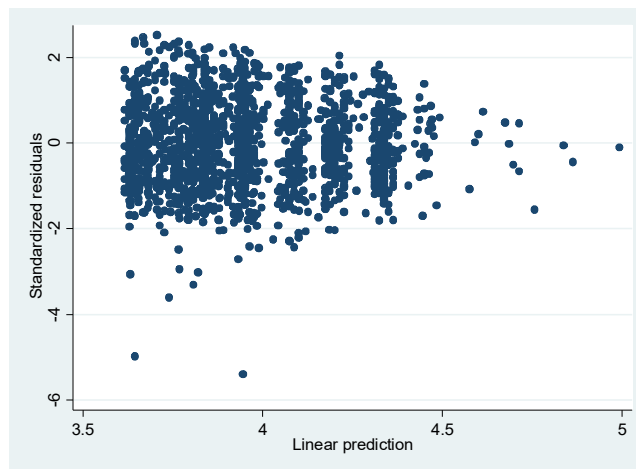
```
Log likelihood = -7960.5368
```

Number of obs	=	1574
LR chi2(8)	=	148.83
Prob > chi2	=	0.000

wpc	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
/theta	.1099104	.0271003	4.06	0.000	.0567947 .1630261

★ value of λ close to 0 \rightarrow log transformation

- homoscedasticity (L1a - p11)

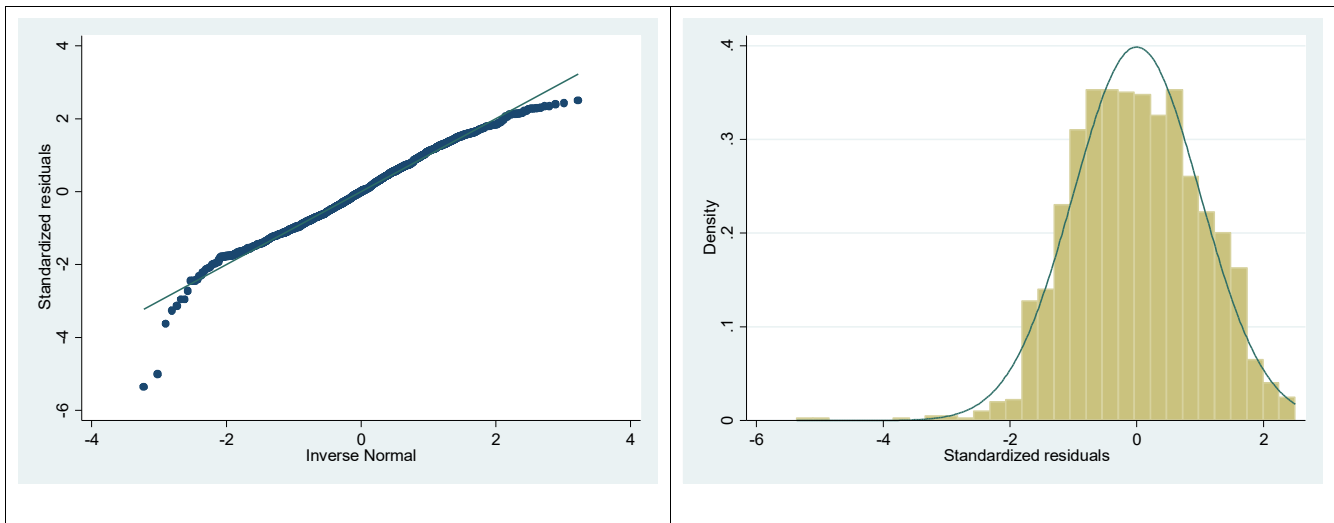


★ tests provide contradicting results

➔ imtest - constant variance

➔ however BPCW test highly significant

● normality (L1a - p10)



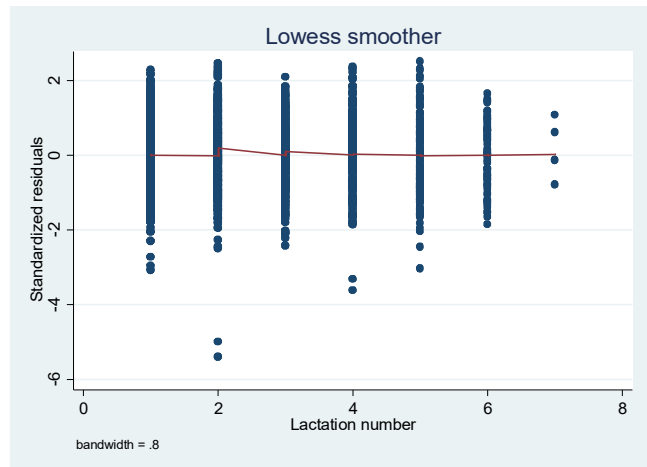
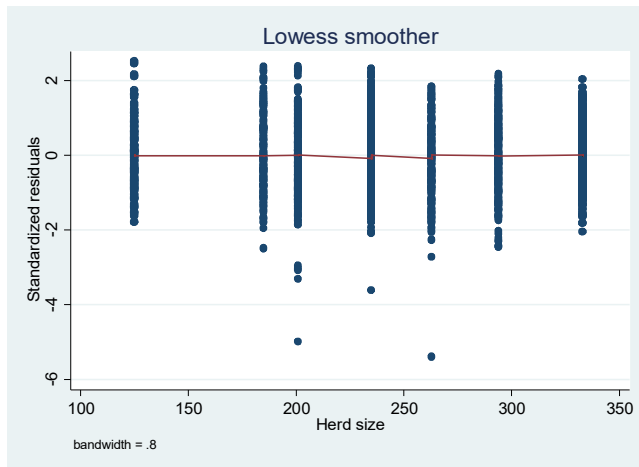
● Linearity (L1a - p10)

★ continuous predictors only

★ standardized residuals vs predictor

Herd size

Parity



★ also pattern can be evaluated by looking at the residuals from a model without the predictor of interest

Evaluating individual observations

Outliers (lecture L1a – pages 8/9)

- large residuals
- standardized residuals
 - ★ expect 5% > 2 and < -2
 - ★ expect 1% > 3 and < -3
- deletion residuals
 - ★ t-test - $\text{prob} = 2 * n * t(\text{DFE}-1, d_i)$
- wpc log-transformed model - residuals
 - ★ standardized residuals
 - expected 5% = 79 (observed = 50)
 - expected 1% = 14 (observed = 6)
 - fewer values than expected
 - ★ deletion residuals
 - outlier cutoff value = 4.17
 - two observations \geq this cutoff point
 - indication that these are outliers
 - cows with wpc = 1
 - delete and refit the model (later)
 - only for diagnostics purposes

Leverage

- x-outliers
 - ★ depend on X values only
 - not affected by relationship with y
 - ★ expresses whether x_i is outlying in distribution of x's
 - so potentially influential
 - ★ less meaningful for categorical predictors
 - depends on the distribution of each category
- cut-off values
 - ★ concerns if $h_i \geq 2(k+1)/n$
 - or $\geq 3(k+1)/n$
 - k =# of predictors, n =# obs.
 - should also look for unusual observations independently of these cutpoints
- Stata - *predict* varname, leverage
 - ★ eg. *predict lev_lnwpc, lev*

● computed after full model (ln_wpc)

. summ lev

Variable	Obs	Mean	Std. Dev.	Min	Max
lev	1574	.0063532	.0083435	.0016636	.0642237

. display "leverage cutoff: " 2*nparam/nobs

leverage cutoff: .01270648

. display "conservative leverage cutoff: " 3*nparam/nobs

conservative leverage cutoff: .01905972

. count if lev>=.01905972 /* many (n=108) high leverage values */

. list wpc wpc_ln fit aut_calv herd_size parity1 twin dyst rp vag_disch stdres lev in 1/10, clean noobs

wpc	wpc_ln	fit	aut_calv	herd_size	parity1	twin	dyst	rp	vag_disch	stdres	lev
76	4.331	4.690	yes	185	4	yes	no	yes	yes	-0.507	0.065
137	4.920	4.989	yes	294	2	yes	no	yes	yes	-0.097	0.064
94	4.543	4.859	yes	263	3	yes	no	yes	yes	-0.445	0.064
45	3.807	4.612	no	201	4	yes	no	yes	yes	-1.135	0.063
53	3.970	4.251	no	263	5	yes	no	no	yes	-0.395	0.059
110	4.700	4.188	no	263	0	yes	no	no	yes	0.720	0.057
32	3.466	4.286	no	201	1	yes	yes	yes	no	-1.150	0.052
99	4.595	4.575	yes	294	1	yes	yes	no	no	0.029	0.051
23	3.135	4.143	no	185	0	yes	yes	no	no	-1.409	0.050
106	4.663	4.600	no	294	0	no	yes	yes	yes	0.088	0.046

. tab3way twin rp dyst

		Dystocia at calving and Retained placenta at calving			
		no		yes	
Twins born		no	yes	no	yes
	no	1332	124	76	15
	yes	15	9	2	1

★ large leverage values for less common combination of categorical predictors

Influence diagnostics: Cook's distance and DFITS

- Cook's distance and DFITS

$$\text{Cooks D} \quad D_i = \frac{r_{si}^2}{(k+1)} * \frac{h_i}{(1-h_i)}$$

$$\text{DFITS} \quad DFITS_i = r_{ti} \sqrt{\frac{h_i}{(1-h_i)}}$$

- ★ measure influence of an observation and have two interpretations:
 - effect of deleting observation "i" on the predictions
 - effect of outlier information about x's and y's
 - ★ D_i is on the squared residual scale and based on standardized residuals
 - ★ $DFITS_i$ is on absolute (signed) residual scale and are based on deletion residuals
- cut-off values

- ★ concern if either is >1 (rare) or if...

- $D_i \geq 1$ or $\geq 4/n$

- $DFITS_i$ outside $\pm 2 * \sqrt{\frac{k+1}{n}}$ (if $n \geq 120$)

- $n < 120$ only values beyond ± 1

- Stata: `predict varname, cooksd/dfits`

● ln_wpc model

★ Cook's

```
. summ cook
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
cook	1574	.0006351	.0013679	5.57e-10	.0174304

```
. display "Cook's D cutoff: " 4/nobs  
Cook's D cutoff: .0025413
```

```
. count if cook>=.0025413 /* many (n=94) high Cook's D values */  
94
```

★ DFITS

```
. summ dfit
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
dfit	1574	.0003931	.0797594	-.4179343	.3664328

```
. display "DFITS cutoff: " 2*sqrt(nparam/nobs)*(nobs>=120)+1*(nobs<120)  
DFITS cutoff: .15941443
```

```
. count if abs(dfit)>=.15941443 /* many (n=92) high DFITS values */  
92
```

★ first lactation cows with reproductive diseases have highest Cook's and DFITS values

Delta-beta (or DFBETA)

★ influence of an observation on a specific regression coefficient (β)

★ for obs. "i" and predictor x_j

$$\rightarrow \text{DFBETA}_{ij} = \frac{(\beta_j - \beta_{j(i)})}{SE_j}$$

→ where $\beta_{j(i)}$ is estimate of β without obs. "i"

★ measures change in β in terms of SE's

→ how many SE's does β_j increase if "i" is dropped?

● cut-off values

★ $\text{DFBETA}_{ij} > 0$ (< 0) ~ obs. "i" pulls β_j up (down)

→ extreme values of DFBETA_{ij} are notable

→ most meaningful for continuous predictors

★ concern if outside $\pm \frac{2}{\sqrt{n}}$ (for $n \geq 120$)

→ $n < 120$ only values beyond +/- 1

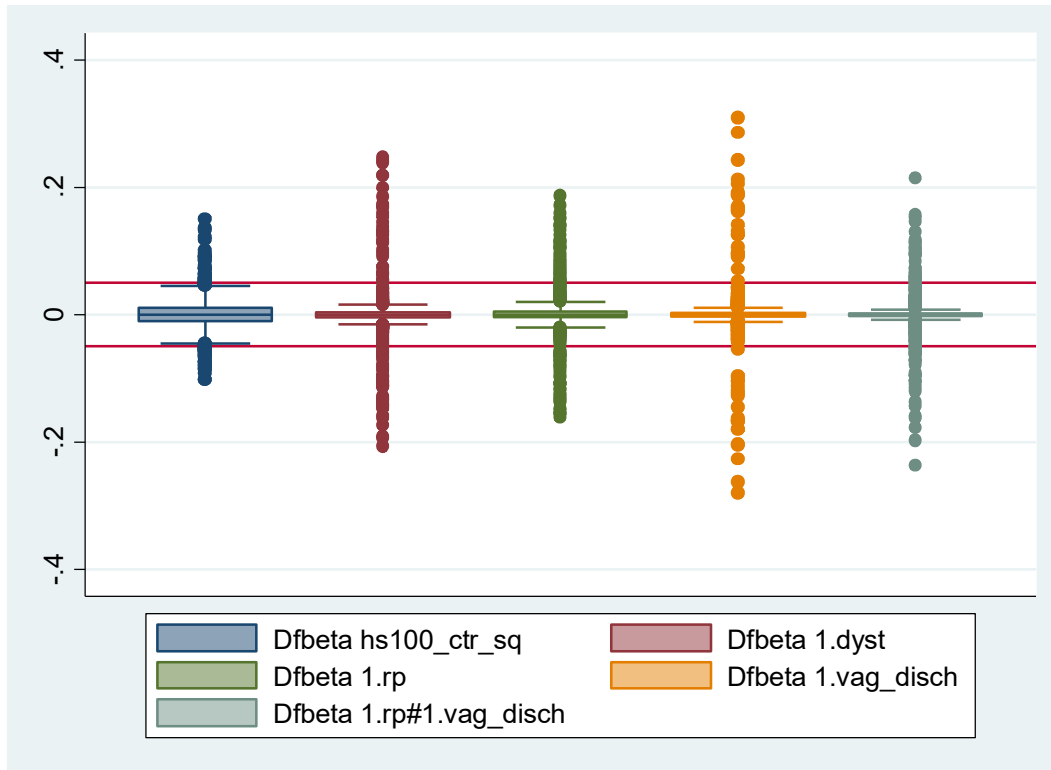
● Stata

★ `predict varname, dfbeta(var in model)`

→ eg. `predict dfbdyst, dfbeta(dyst)`

- ln_wpc model

```
. display "DFBETA cutoff: " 2/sqrt(nobs)* (nobs>=120)+1*(nobs<120)
DFBETA cutoff: .05041127
```



- explore values by browsing/listing
 - ★ large values are cases of the correspondent predictor (eg. cows with dystocia)
 - ★ linear effect of herd size similar to quadratic term
 - ➔ need to be aware of collinearity

What to do with Outliers/Influential observations

- verify that points are not errors (eg. data entry errors)
- may be very informative
 - ★ always examine observations and values of the predictors
- fit models with and without the point(s)
- determine if outliers are significant (L1a -p9)
- keeping them in the analysis
 - ★ potential for biased estimates
 - ★ usually leads to larger SE and reduced power (is therefore conservative)
 - ★ always report them
- eliminating them from the analysis
 - ★ **should be always reported and justified**
 - ★ narrows scope of inference from the study
 - ★ may creates an unrealistically good model

Other transformations

- log-transformed model

- ★ some indication of non-constant variance

- ★ residuals still not normally-distributed

- ➔ two severe outliers

```
. l wpc wpc_ln fit herd_size parity dyst rp vag_disch if wpc==1, clean compress
noobs
```

wpc	wpc~n	fit	her~e	par~y	dyst	rp	vag~h
1	0	3.645548	201	2	no	no	no
1	0	3.945861	263	2	no	no	no

```
. l wpc wpc_ln delres lev dfit cook dfb_dyst dfb_rp dfb_vd dfb_int dfb_hs if wpc==1, clean
compress noobs
```

wpc	wpc~n	delres	lev	dfit	cook	df~st	dfb~p	dfb~d	dfb~nt	dfb hs
1	0	-5.028	0.002	-0.243	0.006	0.051	0.037	0.028	-0.021	0.098
1	0	-5.449	0.002	-0.243	0.006	0.044	0.053	0.042	-0.026	-0.016

- ★ comparison without outliers

```
. estimate table ln ln_noout
```

Variable	ln	ln_noout
hs100_ctr	.34365799	.33916323
hs100_ctr_sq	.21187276	.20269649
parity1	.01298436	.01133333
aut_calv	-.13716214	-.13695147
twin	.39274261	.3894441
dyst	.1109405	.10338895
rp		
yes	.11231661	.10603408
vag_disch		
yes	-.02601346	-.03328151
rp#vag_disch		
yes#yes	.41369992	.42230092
_cons	3.8885867	3.9010244

- ★ repeat but without influential obs.

- ★ advantage log-transformed model

- ➔ interpretation of estimates still possible

- square root transformation
 - ★ example VER - constant variance ->ok
 - ★ residuals still not normally-distributed
 - ★ deletion residuals show no indication of outlying observations
 - ★ interpretation estimates not possible
 - ➔ obtain predicted values
 - ➔ parity=3 aut_calv=1 hsize=0 hsize2=0 twin=0

rp#vag_disch#dyst	Sqrt transformation			log-transformation		
	est	lo_ci	up_ci	est	lo_ci	up_ci
no#no#no	58.267	54.094	62.594	50.127	46.598	53.922
no#no#yes	66.840	63.016	70.776	56.008	52.629	59.604
no#yes#no	58.064	52.980	63.382	48.840	44.659	53.412
no#yes#yes	66.622	65.094	68.168	54.570	53.239	55.934
yes#no#no	64.364	48.202	82.859	56.085	42.254	74.444
yes#no#yes	73.359	63.454	83.983	62.666	53.554	73.327
yes#yes#no	90.259	90.259	90.259	82.645	82.645	82.645
yes#yes#yes	100.857	90.533	111.737	92.342	80.406	106.049

*ci estimated from t-distribution (Stata uses a different estimation procedure)

- ★ sqrt model - better than log-transformed
 - ➔ no outliers (eg. Deletion res. range: -2.42; 3.28)
- ★ influential diagnostics
 - ➔ similar to log-transformed model
 - ➔ more in VER.