Index of Lecture 7: Experimental design

Practical Information

Today's lecture:

- principles of experimental design, with brief overview of ideas and methods,
- a range of classical block designs, including cross-over designs,
- more focus on data analysis than on design construction.[1]

Guidelines for textbook reading:

- skip entirely: discussions of efficiency (13.2.3, 13.3.5, pp. 362_7–368^{11}) and advanced sections (13.5, 14.2-7), as well as refs to Chapters 11-12, incl. Hasse diagrams,
- cross-over designs: GO Sections 13.3.1, 13.3.4, 13.3.6, 16.7.[2]

Other news:

- home assignment #3 (for all) due on Monday (1/3),
- slightly expanded do-file for Lecture 6 posted under lab 6 (Minitab/Stata page).

[1] More detailed references, in particular for design construction:

* Cochran & Cox: *Experimental Designs* (UPEI library),
* Box, Hunter & Hunter: *Statistics for Experimenters* (UPEI library),
* Dean & Voss: *Design and Analysis of Experiments* (UPEI library e-book collections; link via course homepage).

[2] Lecture notes also incorporate material from:

[1] Senn (2005), Crossover Designs, In: *Encyclopedia of Biostatistics* (available electronically at UPEI library),
[2] Toutenburg (2002), *Statistical Analysis of Designed Experiments*, Chapter 9 (also electronic access at UPEI library).

Introduction to Experimental Design

What characterises an experiment?[3]

the experimenter imposes treatments onto the experimental units from which responses are measured.

Advantages of an experiment:

- allows (potentially) to conclude about *causation*,[4]
- can be designed to give small error in comparisons,
- can be designed to avoid/minimise *bias* in comparisons.

Plain glossary for experimental design:

- treatments (short form: tx's): the different procedures to compare,
- experimental units (EUs): the "things/subjects" to which treatments are applied,
- responses: outcomes of interest to compare, observed at EUs after treatments are applied,
- randomisation: random assignment of treatments to experimental units (to avoid unexpected patterns),

[3] As opposed to observational studies.

[4] The reasoning goes: "If experimental units differ only (ideally) by their treatments, any differences beyond random fluctuations in their responses must be caused by the treatments".

- experimental error: variation between equally treated units (both population and observed values),
- measurement units: the actual objects on which responses are measured (may be smaller than the EUs),
- blinding: response evaluators do not know the treatment allocation,
- control: treatment consisting of "no treatment", possibly placebo (inactive treatment),[5]
- factor: (controlled) explanatory, categorical variable,
- level/category: specific value of factor/treatment,
- replication: multiple "identical" experimental units (sometimes, confusingly, repetitions of the entire experiment[6]),[7]
- blocking: division of EUs into homogeneous groups (discussed from 7L−5 onward),
- balancedness = (generally) all treatment levels occur the same number of times,
  - desirable property of a design, but not indispensable,[8]
  - gives same precision on all treatment estimates and comparisons, and simple computational formulae,
- (technical) confounding: the effect of one factor/treatment cannot be distinguished from that of another factor/ treatment (different meaning than in epidemiology!).

[5] The main purpose of a control group is to control lurking variables/confounders.
[6] Repetition of an entire experiment would usually be considered a blocking factor.
[7] The purpose of replication is to enable estimation of experimental error.
[8] Balancedness is necessary for some ANOVA methods − however, (general) linear model methods always apply.

A Checklist for Planning Experiments

(from Dean & Voss, Chapter 2, pages 7–14)[9]

(a) Define the objectives of the experiment.

(b) Identify all sources of variation, including:

  (i) treatment factors and their levels,

  (ii) experimental units,

  (iii) blocking factors, noise factors, and covariates.

(c) Choose a rule for assigning the experimental units to the treatments (the narrow meaning of experimental design).

(d) Specify the measurements to be made, the experimental procedure, and the anticipated difficulties.

(e) Run a pilot experiment.

(f) Specify the (statistical) model.

(g) Outline the (statistical) analysis.

(h) Calculate the number of observations that need to be taken.

(i) Review the above decisions. Revise if necessary.

[9] Chapter 2 is also available at the VHM 802 Moodle account.

Block Design versus Completely Randomised Design

Blocks = groups of homogeneous experimental units (EUs), so that

- EUs are more alike within than between groups, before and during experiment, possibly because:
  - EUs within blocks share inherent characteristics that cannot be assigned to them (e.g., gender, location).

Contrasting block and completely randomised designs:[10]

Characteristic	Block design	Completely randomised design
Randomization of tx's	within EUs per block	among all EUs
Replication of tx's	possible, not necessary ≤ 1 or ≥ 1 EU per tx per block	necessary > 1 EU per tx
Error variance	among EUs per block (if no tx's)	among all EUs (if no tx's)
Balancedness	different meanings, simplest: same # EU per tx per block	same # EU per tx
Model impact	include block effects	none

How to distinguish completely randomized and block designs?
— (GO) follow the randomisation!

[10] Basic introductions to the two designs were given in Lecture 2 of VHM 801.

Statistical Analysis of Block Designs

Statistical model for observations (y_{ij}) on treatments ($i = 1, \ldots, g$) in blocks ($j = 1, \ldots, r$):

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \quad \text{where}$$

- α_i and $\beta_j \sim$ effects of treatment i and block j, respectively,
- the errors $\varepsilon_{11}, \ldots, \varepsilon_{gr}$ are i.i.d. and $\sim \mathrm{N}(0, \sigma^2)$.
- block effects are (usually) additive to treatments:
  - assumes that tx effects do not depend on blocks ("should" be true?),
  - most useful when tx by block interaction believed to be absent or weak,[11]
  - non-additive block effects: only possible with replicated treatments within blocks or a factorial treatment structure (and omitted interactions).

Statistical analysis follows usual ANOVA/linear model principles, except that

- block effects often of less interest (for interpretation),
- blocks are usually not randomized $\Rightarrow$ (GO) tests for block effects could be omitted entirely,
- possible to compute efficiency of block design relative to completely randomized design ("did the block design work?"), see GO for details (not part of course).

[11] Ignoring an existing tx by block interaction $\Rightarrow$ inflated σ^2 and loss of power.

(Unreplicated) Block Design – Example

Mealybug example: (GO Example 13.1)

- changes (before minus after treatment) in counts of mealybugs,
- 3 treatments (water, spores, oil), 5 plants ($\sim$ blocks),
- data layout:
- experimental unit:
  branches of cycad plants,
- measurement units:
  two patches per branch
  treated the same way,

Treatment \ Plant	1	2	3	4	5
water	-9	18	10	9	-6
	-6	5	9	0	13
spores	-4	29	4	-2	11
	7	10	-1	6	-1
oil	4	29	14	14	7
	11	36	16	18	15

- outcome: average change across the two patches per branch.

ANOVA table:

Source	DF	SS	MS	F	P-value
Treatments	2	432.0	216.0	12.2	.004
Plants	4	686.4	171.6	(9.7)	(.004)
Error	8	141.8	17.7		
Total	14	1260.2			

* clear treatment effects:
  oil treatment has larger
  change than the others:
  water $(4.3)^b$, spores $(5.9)^b$, oil $(16.4)^a$,
* plant effects also clearly significant (but less important).

Latin Square Design

- definition: designs with
  - * g treatments,
  - * two blocking variables,
  - * g^2 experimental units arranged in a square,

  and every treatment exactly once in each row / column,
- examples: (cyclic Latin squares, plus permutations)

$g=2$:

A	B
B	A

B	A
A	B

$g=3$:

A	B	C
B	C	A
C	A	B

A	C	B
C	B	A
B	A	C

$g=4$:

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

- advantage: takes into account two blockings
  $\Rightarrow$ potentially more accurate (lower experimental error),
- disadvantage: specific requirements for dimensions of design (number of blocks, number of experimental units),
- randomisation between squares: not easy to do properly[12];
  — a simple and "crude" method randomly permutes rows, columns and symbols,
- modelling: additive effects of rows and columns,
- balanced in the treatments $\Rightarrow$ simple analysis (due to orthogonal factors).

[12] Fisher-Yates procedure based on standard squares tabled, see GO p. 327.

Latin Square Data Example

Mangold (wurzel) example: (Mercer & Hall 1911; field trial)

- yield of plots located in a 5×5 Latin square and with 5 (unspecified) treatments,

$$\begin{aligned} y_{ijk} &= \text{weight of mangold in plot with treatment } i \text{ at location } (j,k), \\ i &= \text{A,B,C,D,E} \sim \text{treatments}, \\ j,k &= 1,2,3,4,5 \sim \text{rows and columns}, \end{aligned}$$

  technical note: only some sets (i,j,k) occur,

- data layout:
  (treatments and yields)

D 376	E 371	C 355	B 356	A 335
B 316	D 338	E 336	A 356	C 332
C 326	A 326	B 335	D 343	E 330
E 317	B 343	A 330	C 327	D 336
A 321	C 332	D 317	E 318	B 306

- statistical model (additive treatments and blocks):

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk},$$

  where the ε_{ijk}'s are i.i.d. and $\sim \mathrm{N}(0, \sigma^2)$.

Note: Latin squares can be extended to allow for two treatments (and still two blocks)
$\rightarrow$ Graeco-Latin squares (GO Section 13.4).

Mangold Data Results

ANOVA table
for additive model:

Source	DF	SS	MS	F	P-value
Rows	4	4240.2	1060.1	(7.25)	(.003)
Columns	4	701.8	175.5	(1.20)	(.36)
Treatments	4	330.2	82.6	0.56	.69
Error	12	1754.3	146.2		
Total	24	7026.6			

- no indication of treatment effects, but D highest; we explore contrast comparing D to average of others:

$$\begin{aligned}
\hat{w} &= 4\hat{\alpha}_D - \hat{\alpha}_A - \hat{\alpha}_B - \hat{\alpha}_C - \hat{\alpha}_E \\
&= 4\bar{y}_{D\cdot\cdot} - \bar{y}_{A\cdot\cdot} - \bar{y}_{B\cdot\cdot} - \bar{y}_{C\cdot\cdot} - \bar{y}_{E\cdot\cdot} = 34.4, \\
\mathrm{SE}(\hat{w}) &= \sqrt{\mathrm{MS}_E\,(4^2+1^2+1^2+1^2+1^2)/5} = 24.18, \\
t &= \hat{w}/\mathrm{SE}(\hat{w}) = 34.4/24.2 = 1.42 \quad \sim \text{ clearly NS even if pre-planned,} \\
\mathrm{SS}(\hat{w}) &= t^2\,\mathrm{MS}_E = 295.84 \quad \sim \text{ 90\% (295.84/330.24) of variation,}
\end{aligned}$$

- no column effects, but clear row effects:
  row 1 > rows 2−4 > row 5 (maybe not particularly interesting).

Further analyses:

- model reduction: possible to drop non-significant effects, but not necessary,
- diagnostics: all look good.

Incomplete Block Designs

Incomplete: the blocks do not comprise all treatments (combinations of tx factors).

Why of potential interest?

- maybe limited block sizes (e.g., litters or twins),
- maybe save experimental units (economy),
- missing values ⇒ incomplete blocks.

Main points about incomplete block designs:

- unbalanced (in an extreme form) ⇒ order of testing for factors is important: partial SS ≠ sequential SS,[13]
- "balanced" incomplete block design (BIBD): balanced in the sense that all pairs of treatments occur equally often in the same block ⇒ same precision on all treatment comparisons,
- many specialised designs exist with certain properties (GO 14.2–7; not in course),
- statistical analysis: using methods for (general) linear models, in particular least squares means.[14]

[13] Recall that partial/adjusted SS ∼ removing effect while keeping all others, and sequential SS ∼ removing effects sequentially (bottom up in ANOVA table).

[14] Recall that least squares means are adjusted for other factors by giving their levels/categories equal weights; this would seem the correct approach when unequal representation occurs by design.

Balanced Incomplete Block Design (BIBD)

A BIBD with g treatments and b blocks must meet the requirements,

- all treatments occur the same number of times, r,
- all blocks are of the same size, k,
- every pair of treatments "meet" the same number of times, λ,

Then the following relations hold:

$$rg = bk \quad \text{and} \quad \lambda(g-1) = r(k-1).$$

Construction of BIBD's:

- designs do not always exist (even if above relations satisfied),
- designs tabled in textbooks, e.g. GO or Cochran & Cox.

Statistical model has additive treatment and block effectss:

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \quad i = 1, \ldots, g \ (\sim \text{treatments}), \quad j = 1, \ldots, b \ (\sim \text{blocks}),$$

and where not all pairs (i, j) occur.

Technical notes on analysis:

- treatment SE computable by hand, using "effective sample size" ($\lambda g/k$) instead of r,
- "intrablock" analysis $\sim$ linear model; "interblock" analysis $\sim$ random effects model.

BIBD Examples

Simplest BIBD:
($g=3$, $b=3$, $r=2$, $k=2$, $\lambda=1$):

Block	1	2	3
Treat-	A	B	C
ment	B	C	A

Dish detergent example: (GO Example 14.2)

- count plates before foam disappears, in "sessions" with 3 simultaneous operators and 3 basins,

  y_{ij} = number of plates for detergent i in session j
  i = A,B,C,D,E,F,G,H,J $\sim$ detergents, treatments
  j = $1,\ldots,12$ $\sim$ sessions, blocks,

- experimental layout:

Block	1	2	3	4	5	6	7	8	9	10	11	12
Treat-	A	D	G	A	B	C	A	B	C	A	B	C
ment	B	E	H	D	E	F	E	F	D	F	D	E
	C	F	J	G	H	J	J	G	H	H	J	G

- BIBD with $g=9$, $b=12$, $r=4$, $k=3$, $\lambda=1$,
- further treatment structure:
  - A,B,C,D $\sim$ detergent I + doses 3,2,1,0 of additive,
  - E,F,G,H $\sim$ detergent II + doses 3,2,1,0 of additive,
  - J $\sim$ control.

Introduction to Cross-Over Designs

Definition of a cross-over trial:

> "a trial in which individual subjects are given sequences of treatments with the object of studying differences between individual treatments (or subsequences of treatments)" [1]

- idea: each subject forms a block (or one's own control)
  $\Rightarrow$ reduced variation (by eliminating between-subject variation),
- advantages:
  - (potentially) (much) higher efficiency than completely randomized design,
  - individuals' reactions to treatments can be studied,
- drawbacks:
  - longer execution time $\Rightarrow$ logistical challenges, greater risk of subject dropouts,
  - more complex data analysis (due to added risk of bias caused by carry-over effects), and really a special case of repeated measures,
- simple $\mathbf{3 \times 3}$ Latin square example: Bioequivalence (GO Example 13.6).

Common terminology:

- periods: occasions where subjects are treated (and measured),
- carry-over effect: residual effect from one period to the next,
- wash-out period: interval between tx periods to eliminate/reduce carry-over effects.

Simplest Design: AB/BA

- two treatments (A and B),
- two periods (1 and 2),
- two groups of subjects:

	Period 1	Period 2
subject group 1	A	B
subject group 2	B	A

Data example: Scents and learning[15] — can pleasant (floral) aromas help a student learn better?

- completion times for pencil and paper mazes for 21 subjects with scented (S) and unscented (U) masks,
- 11 subjects used unscented masks first, 10 subjects scented masks first (1 subject excluded):

Subject	Sequence	mean U	mean S	diff U−S	diff 1−2
1	US	30.6	38.0	-7.4	-7.4
2	SU	48.4	51.6	-3.2	3.2
3	US	60.8	56.7	4.1	4.1
4	SU	36.1	40.5	-4.4	4.4
5	US	68.5	49.0	19.5	19.5
6	SU	32.4	43.2	-10.8	10.8
...	...	...	...		

[15] Reduced data set (Mazes and smells) available at: `dasl.datadescription.com`.

Analysis of AB/BA Design

Assuming no carry-over effects, simple approaches will work:

- compare treatments by two-sample analysis (e.g. t-test) for differences $1-2$,[16]
- compare periods by two-sample analysis (e.g. t-test) for differences A−B.

Combined analysis of response y_{ijk} in period j for k'th subject with treatment sequence i, using the model

$$y_{ijk} = \mu_{ij} + s_{ik} + \varepsilon_{ijk}, \qquad \text{where}$$

* μ_{ij} is the mean for sequence i in period j, given by:

sequence i	period 1 ($j=1$)	period 2 ($j=2$)
AB	$\mu + \alpha_A + \beta_1$	$\mu + \alpha_B + \beta_2 + \lambda_{AB}$
BA	$\mu + \alpha_B + \beta_1$	$\mu + \alpha_A + \beta_2 + \lambda_{BA}$

  $\alpha_A, \alpha_B \sim$ treatment effects; $\beta_1, \beta_2 \sim$ period effects; $\lambda_{AB}, \lambda_{BA} \sim$ carry-over effects,

* s_{ik} is the effect of subject k with treatment sequence i,
* ε_{ijk} is the error term $\sim \mathrm{N}(0, \sigma^2)$,
* same inference as above if $\lambda_{AB} = \lambda_{BA} = 0$,
* complex analysis ([1],[2]) if λ's $\neq 0$.

[16] Treatment comparison by a paired two-sample analysis for measurements for A and B (effectively a one-sample analysis for differences A−B) is only valid when no period effects exist.

More Treatments and Periods

Examples of extensions of AB/BA design:

- 2 tx, $>$ 2 periods: enables modelling of carry-over effects in analysis, e.g. for sequences AABB and BBAA:

	Sequence				Sequence			
Carry-over model	A	A	B	B	B	B	A	A
"prev. period"	–	λ_A	λ_A	λ_B	–	λ_B	λ_B	λ_A
"change only"	–	0	λ_{AB}	0	–	0	λ_{BA}	0
"prev. + present"	–	λ_{AA}	λ_{AB}	λ_{BB}	–	λ_{BB}	λ_{BA}	λ_{AA}

- 3 tx, 3 periods: to ensure balancedness of tx's in periods traditionally laid out in Latin squares, e.g.,

A	B	C
B	C	A
C	A	B

A	C	B
C	B	A
B	A	C

  where rows $\sim$ periods, columns $\sim$ subjects, symbols $\sim$ tx,

  * simple analysis when no carry-over effects,
  * desirable to include all sequences by combining two different Latin squares (as shown above),

- g tx, g periods: use (multiple) $g \times g$ Latin squares,
- g tx, $k < g$ periods: use incomplete block design, preferably BIBD (g, b, k, r, λ).

Combining Latin Squares

Multiple Latin squares in same design:

- increases the error degrees of freedom ⇒ larger power,
- several extra options for modelling, depending on data context.

Notation/Model:

$$\begin{cases} y_{ijkl} = \text{ outcome for tx } i \text{ in row } j \text{ and column } k \text{ in square } l, \\ y_{ijkl} = \mu + \alpha_i + \beta_{j(l)} + \gamma_{k(l)} + \varepsilon_{ijkl}, \end{cases}$$

- row ($\beta_{j(l)}$) and column ($\gamma_{k(l)}$) effects "nested in" (separate for) squares ∼ different effects across squares,
- a block effect may be assumed the same in all squares, e.g. for periods:
  - same effects if all subjects go through same periods,
  - different effects if periods are not the same (e.g. due to different ages of subjects),

Modelling refinements:

- square type[17] interactions: carry-over effects may show up as interactions between square type and periods,
- residual effects may be modelled directly to split each tx effect into "direct" and "residual" effects.

[17] The square type is determined by the carry-over combinations it contains.

Latin Square Cross-Over Trial Examples

Bioequivalence trial (GO Example 13.10) with 12 subjects in 4 Latin squares.

Milk production example $\sim$ cross-over trial (GO Example 13.12):

- milk yield of cows during three periods with different diets,

  y_{ijkl} = yield for cow k in square l in period j on diet i
  i = A,B,C $\sim$ diets (roughage, limited grain, full grain)
  j = $1, 2, 3 \sim$ period (for each cow),
  k = $1, \ldots, 3 \sim$ cow number (within squares)
  l = $1, \ldots, 6 \sim$ Latin square number.

	Cow			Cow			Cow		
Period	1	2	3	7	8	9	13	14	15
1	A	B	C	A	B	C	A	B	C
2	B	C	A	B	C	A	B	C	A
3	C	A	B	C	A	B	C	A	B

	Cow			Cow			Cow		
Period	4	5	6	10	11	12	16	17	18
1	A	B	C	A	B	C	A	B	C
2	C	A	B	C	A	B	C	A	B
3	B	C	A	B	C	A	B	C	A

- 6 separate Latin squares,
  - two types of Latin squares (top/bottom), 3 replicates of each,
  - top $\sim$ diet order AB, BC, CA; bottom $\sim$ AC, CB, BA,
- basic statistical model (additive, no square effects),

$$\begin{aligned} y_{ijkl} &= \mu + \alpha_i + \beta_j + \gamma_{k(l)} + \varepsilon_{ijkl}, \text{ or} \\ y_i &= \mu + \alpha_{\text{diet}(i)} + \beta_{\text{per}(i)} + \gamma_{\text{cow}(i)} + \varepsilon_i, \end{aligned}$$

where the errors are i.i.d. and $\sim \text{N}(0, \sigma^2)$.